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The suppression of a Bloch band in a driving laser field

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Abstract. The dynamic effect of Bloch electrons in a spatial periodic system under the influence of a driving laser field is studied within the single-band approximation. A quasienergy band is obtained exactly for the case of long-range intersite interactions. It is found that the quasienergy band will be suppressed heavily by the laser field if the ratio of the Bloch frequency to the laser frequency is a root of the ordinary Bessel function of order zero. If only the nearest-neighbour intersite interaction is involved, the band suppression will turn into the band collapse proposed very recently. However, a numerical calculation of the band width shows that in comparison with the nearest-neighbour intersite interaction, the contribution of other intersite interactions to the quasienergy is not negligible. This may imply that the experimental observation of the dynamic effect will be that of band suppression rather than band collapse.

Considerable attention has been devoted to the dynamic effect of Bloch electrons in a periodic potential under the influence of an external electric field. It is well known that the energy states for such a periodic system with an applied DC electric field would form equally spaced discrete levels, the so-called 'Stark ladder' [1], and the corresponding wave functions would be localized; this is referred to as 'Bloch oscillation' [2]. The experimental observation of the Stark ladder or Wannier–Stark localization (WSL) in solids, as well known, suffers certain difficulties because the required electric field is too high [3]. However, this situation has been changed in semiconductor superlattices, where the width of the miniband is reduced as a result of the large periodicity [4]. Therefore, the required field strength is also reduced, and the WSL turns out to be observable [5].

The motion of an electron moving in a spatial periodic system with an applied AC electric field shows more fascinating aspects. In the investigation of the evolution of the behaviour of an electron in such a system with the tight-binding approximation, it is found that an initially localized electron will remain localized if the ratio of the field magnitude to the field frequency is a root of the ordinary Bessel function of order zero [6]. This new phenomenon involving the dynamic localization of the moving carrier is quite interesting. Therefore, it is worthwhile to study the quasienergy in such systems.

In this paper, we address our novel results on this aspect. We find that for a general setting where the Bloch electron in a spatial periodic system is subjected to a driving laser field, a quasienergy band can be derived from Floquet's theorem [7] analytically and explicitly even if long-range intersite interactions are taken into account. It is found that this quasienergy band will be suppressed heavily by the driving laser field if the ratio of the Bloch frequency to the laser frequency is a root of the ordinary Bessel function of order zero. If only the nearest-neighbour intersite interaction is involved, such a band suppression will turn into the band collapse proposed very recently [8]. However, our numerical calculations

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show that in comparison with the nearest-neighbour intersite interaction, the contribution of other intersite interactions to the quasienergy is not negligible. This may mean that the experimental observation of the dynamic effect will be that of band suppression rather than band collapse.

For the sake of simplicity, we consider an electron in one-dimensional systems with a spatial periodic potential under the influence of a driving laser field. The Hamiltonian is

$$H(x,t) = H_0(x) - eEx\cos(\omega t) \tag{1}$$

where $H_0(x)$ is the field-free part of the Hamiltonian and is spatially periodic with a period a, $H_0(x) = H_0(x+a)$, e and E are the charge and the strength of the laser field, respectively. The Hamiltonian (1) is also periodic in time, $H(x,t) = H(x,t+T)$, where $T = 2\pi/\omega$ is the length of an optical cycle. Hence, Floquet's theorem asserts that the evolution wave functions of $H(x,t)$ can be written as $\psi(x,t) = \exp(-\mathrm{i}\epsilon t)u_\epsilon(x,t)$ with $u_\epsilon(x,t) = u_\epsilon(x,t+T)$, where ϵ is the quasienergy. The T-periodic functions $u_\epsilon(x,t)$ satisfy the Schrödinger equation

$$\left[H(x,t) - \mathrm{i}\frac{\partial}{\partial t}\right]u_\epsilon(x,t) = \epsilon u_\epsilon(x,t). \tag{2}$$

From the dynamical point of view, the process of Zener tunneling is sufficiently slow to be considered unimportant [9]. Therefore we only pay attention to the single-band case. The wave functions $u_\epsilon(x,t)$ can be expressed as a linear superposition of single-band Wannier functions

$$u_\epsilon(x,t) = \sum_n u_{\epsilon n}(t)\phi(x-na) \tag{3}$$

with $u_{\epsilon n}(t) = u_{\epsilon n}(t+T)$, where $\phi(x-na)$ is the Wannier function at site n. The amplitudes $u_{\epsilon n}(t)$ satisfy the evolution equations

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}u_{\epsilon n}(t) = \sum_m \langle 0|H|m\rangle u_{\epsilon(m+n)}(t) - [\epsilon + neaE\cos(\omega t)]u_{\epsilon n}(t) \tag{4}$$

with

$$\langle 0|H|m\rangle = \int_{-\infty}^{\infty}\mathrm{d}x\,\phi^*(x)H(x,t)\phi(x-ma). \tag{5}$$

Introducing

$$v_{\epsilon n}(t) = \exp\{-\mathrm{i}[\epsilon t + n(eaE/\omega)\sin(\omega t)]\}u_{\epsilon n}(t) \tag{6}$$

(4) reads

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}v_{\epsilon n}(t) = \sum_m \langle 0|H|m\rangle \exp[\mathrm{i}m(eaE/\omega)\sin(\omega t)]v_{\epsilon(m+n)}(t). \tag{7}$$

The discrete Fourier transform

$$V_{\epsilon k}(t) = \sum_n v_{\epsilon n}(t)\exp(-ink) \qquad (0 \leqslant k < 2\pi) \tag{8}$$

leads to the solutions of (7) being

$$V_{\epsilon k}(t) = V_{\epsilon k}(0)\exp\Big[-\mathrm{i}\int_0^t \mathrm{d}t' H(k,t')\Big] \tag{9}$$

where

$$H(k,t) = \sum_m \langle 0|H|m\rangle \exp\{\mathrm{i}m[k + (eaE/\omega)\sin(\omega t)]\}. \tag{10}$$

Note that from (6), we have

$$v_{\epsilon n}(t+T) = v_{\epsilon n}(t)\exp(-\mathrm{i}\epsilon T) \tag{11}$$

because $u_{\epsilon n}(t) = u_{\epsilon n}(t+T)$. Therefore, (8) and (11) yield

$$V_{\epsilon k}(t+T) = V_{\epsilon k}(t)\exp(-\mathrm{i}\epsilon T). \tag{12}$$

On the other hand, since $H(k,t)$ is also periodic in time with the period T, $H(k,t) = H(k,t+T)$, (9) gives

$$V_{\epsilon k}(t+T) = V_{\epsilon k}(t)\exp\Big\{-\mathrm{i}\int_0^T \mathrm{d}t\, H(k,t)\Big\}. \tag{13}$$

By comparing (12) and (13), we find that the quasienergy is

$$\epsilon(k) = \frac{1}{T}\int_0^T \mathrm{d}t\, H(k,t)\ \mathrm{mod}(\omega). \tag{14}$$

By substituting (10) into (14), we obtain the final result for $\epsilon(k)$ in the first quasienergy Brillouin zone $(-\pi/T, \pi/T)$ as

$$\epsilon(k) = \sum_{n=-\infty}^{+\infty} R_n J_0(neaE/\omega)\mathrm{e}^{\mathrm{i}nk} \tag{15}$$

where $R_n = \langle 0|H_0(x)|n\rangle$, and J_0 is the ordinary Bessel function of order zero. Note that the condition $-\pi/T \leqslant \epsilon(k) < \pi/T$ can always be met through the choice of the hopping matrices R_n, therefore we ignore it in following discussions. Assuming $R_n = R_{-n}$, (15) yields

$$\epsilon(k) = R_0 + 2\sum_{n=1}^{\infty} R_n J_0(neaE/\omega)\cos(nk). \tag{16}$$

It is well known that the energy band of the undriven system is

$$\epsilon_0(k) = R_0 + 2\sum_{n=1}^{\infty} R_n \cos(nk). \tag{17}$$

Therefore, by comparing (16) and (17) we find that the influence of the driving laser field on the quasienergy band is to suppress the band width through the effective hoppings $R_n^{\mathrm{eff}} = R_n J_0(neaE/\omega)$ because of the decay of Bessel function J_0 with the increase of its

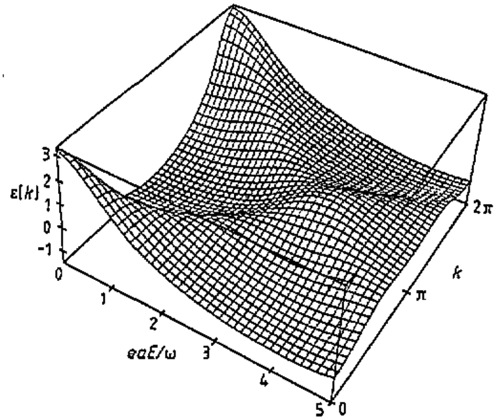

Figure 1. The reduced quasienergy band $\varepsilon(k) = [\epsilon(k) - R_0]/R_1$ plotted versus the function eaE/ω and the dimensionless wave vector k with the approximation of 50 neighbour intersite interactions.

argument. This observation has been illustrated in figure 1, where the reduced quasienergy band $\varepsilon(k) = [\epsilon(k) - R_0]/R_1$ is plotted versus the function eaE/ω and the dimensionless wave vector k. In our calculations, we choose the hopping matrix elements $R_n = R_0 e^{-n}$, and take the long-range intersite interactions up to 50 neighbour sites. This describes an array of 100 identical quantum wells of a superlattice with period a. It is clearly seen from figure 1 that the band width of the reduced quasienergy decreses rapidly if the laser field is switched on.

When only the approximation of the nearest-neighbour intersite interaction is involved, the quasienergy band reads

$$\epsilon(k) = R_0 + 2R^{\text{eff}} \cos k \tag{18}$$

where

$$R^{\text{eff}} = R_1 J_0(eaE/\omega). \tag{19}$$

Equation (19) shows a remarkable result that the effective hopping R^{eff} vanishes entirely whenever the ratio of the Bloch frequency $\Omega = eaE$ and the laser frequency ω is a root of J_0 . Thus, the band width becomes zero, and the quasienergy becomes exactly the on-site energy R_0. This phenomenon of band collapse has already been found in [8], and is consistent with the dynamic localization for moving carriers in AC fields [6]. However, we should notice that such a band collapse is only a consequence of the approximation of the nearest-neighbour intersite interaction. If the long-range intersite interactions are taken into account, it will disappear. This conclusion stems from the observation of (16), where we find that $J_0(neaE/\omega)$ cannot simultaneously vanish for all n since the roots of J_0 are not spaced equidistantly on the real line. These theoretical analyses have been illustrated in figures 2 and 3. Figure 2 shows the appearance of the band collapse (the case of the nearest-neighbour intersite interaction), where it is easy to see that the band width of the reduced quasienergy $\varepsilon(k) = 2J_0(eaE/\omega)\cos k$ becomes zero when $eaE/\omega = 2.4048$, the first root of J_0. In contrast to this situation, the reduced quasienergy maintains a finite band width at the collapse point of $eaE/\omega = 2.4048$ when the next-neighbour intersite interactions are involved (figure 3). The direct calculation shows that when $eaE/\omega = 2.4048$, the band width of the reduced quasienergy for the case of next-neighbour intersite interactions is $\Delta\varepsilon = 0.35$ (see table 1). In fact, the band width can be calculated numerically for any range of intersite interactions and any value of eaE/ω , which yields non-zero band width for all cases except the approximation of the nearest-neighbour intersite interaction. This confirms again that the band collapse cannot occur in the driven system with variable-range hopping matrix elements. However, note that among the hopping matrix elements R_n $(1 \leqslant n < \infty)$, R_1 is dominant in most systems, a significant reduction in the band width will appear when eaE/ω is a root of J_0. Other reductions in the band width, which are obviously not significant, will occur one by one when $neaE/\omega$ $(n = 2, 3, \cdots)$ are roots of J_0. It is these characteristics that constitute figure 1.

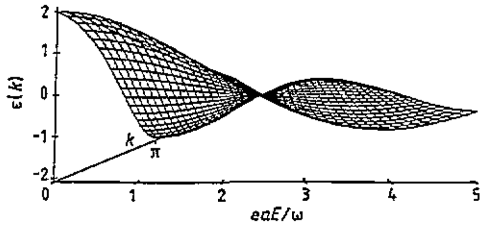

Figure 2. The appearance of the band collapse; only the nearest-neighbour intersite interaction is taken into account.

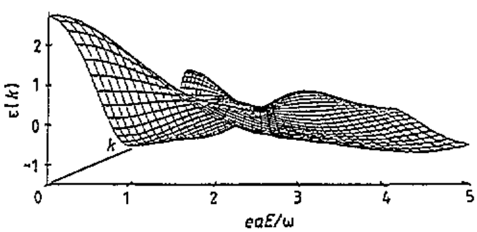

Figure 3. The disappearance of the band collapse when the next-neighbour intersite interactions are involved.

Table 1. The reduced band width of the undriven system ($\Delta\varepsilon_0$) and the driven system ($\Delta\varepsilon_E$, when $eaE/\omega = 2.4048$) corresponding to N-neighbour intersite interactions; $r = \Delta\varepsilon_E/\Delta\varepsilon_0$.

N	$\Delta\varepsilon_0$	$\Delta\varepsilon_E$	r
1	4	0	0
2	4.15	0.35	0.08
5	4.61	0.47	0.10
10	4.63	0.47	0.10
50	4.63	0.47	0.10
∞	4.63	0.47	0.10

To form a quantitative picture of the effect of band suppression in the driven system, we list, corresponding to the N-neighbour intersite interactions, the reduced band width of the undriven system ($\Delta\varepsilon_0$) and its first significant reduction in the driven system ($\Delta\varepsilon_E$, when $eaE/\omega = 2.4048$, the first root of J_0) as well as the ratio $r = \Delta\varepsilon_E/\Delta\varepsilon_0$, which describes the band suppression by the driving laser field, in table 1. Here, we still take the hopping matrix elements $R_n = R_0 e^{-n}$. As shown in table 1, after the first significant reduction, we have still $r \simeq 10\%$ if the variable-range hopping matrix elements are taken into account ($N \geqslant 2$). This means that the contribution to the quasienergy by the long-range intersite interactions is not negligible.

In summary, we have investigated the dynamic effect of Bloch electrons in a spatial periodic potential under the influence of a driving laser field within the single-band approximation. The quasienergy band has been obtained exactly for the case of long-range intersite interactions. It is found that the quasienergy band will be heavily suppressed by the driving laser field if the ratio of the Bloch frequency to the laser frequency is a root of the ordinary Bessel function of order zero. The analysis of the spectrum shows that the band collapse arises merely from the approximation of the nearest-neighbour intersite interaction, and the contribution from the long-range intersite interactions is not negligible. This implies that the observation more likely fulfilled in experiments is the dynamic effect of band suppression rather than that of band collapse.

In our treatment of the band suppression, the effect of electron scattering has not been considered. This approximation is valid provided the scattering time is long compared to the period of the laser field. If such a condition cannot be met in practice, the role of electron scattering should be involved in the description of electron dynamics.

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